Dataset Management in Data-Enabled Predictive Control

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Research field Data-Driven (*Data-EnablEd*) Predictive Control (*DeePC*), *the behavioral approach*

- Fundamental Lemma reformulations
- Dataset management ⟵ this talk
- Computational efficiency

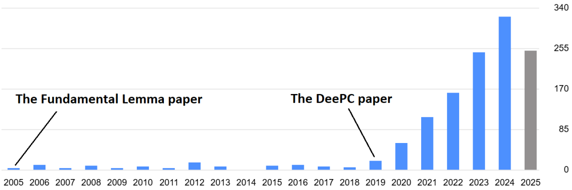
We have several questions to discuss

1. What are the behavioral approach and DDPC/DeePC for LTIs?

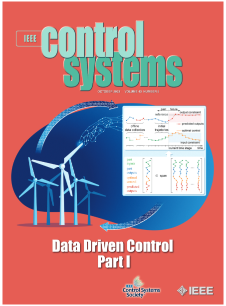
2. How can it be applied to nonlinear systems and what is the dataset management problem?
Examples included

3. What is the proposed solution?

The DDPC was motivated by the Fundamental Lemma



Citations of the Fundamental Lemma paper



Applications

- Quadcopters
- Synchronous drives
- Power converters
- Climate control
- Robotic arm

A dynamical system is a set of trajectories

Models (representations): transfer function, state-space...

Behavior

Let $w = \begin{bmatrix} u \\ y \end{bmatrix} : \mathbb{Z} \to \mathbb{R}^{n_u+n_y}$ be a trajectory. The behavior is the set of all possible trajectories:

$$\mathcal{B} = \{w \,|\, \exists x : \mathbb{Z} \to \mathbb{R}^n \text{ s.t. } \sigma x = Ax + Bu,\ y = Cx + Du\}, \quad \sigma x_k = x_{k+1}$$

or

$$\mathcal{B} = \{w \,|\, A(\sigma)y = B(\sigma)u\} \quad \text{or} \quad \mathcal{B} = \{w \,|\, R(\sigma)w = 0\}$$

Restriction of the trajectory w on the interval $\big[1, L\big]$: $w|_L = \big(w_1, \quad \ldots \quad w_L\big)$

- For L large enough, the restricted $\mathcal{B}|_L$ specifies the whole $\mathcal{B}$

The data must be exciting to learn the behavior

Definition (Excitation)

Let z be a q-variate signal with N samples. We say that z is persistently exciting of order L if the $(qL) \times (N - L + 1)$ Hankel matrix

$$\mathcal{H}_{[L,N]}(z) = \begin{bmatrix} z_1 & z_2 & z_3 & \cdots & z_{N-L+1} \\ z_2 & z_3 & z_4 & \cdots & z_{N-L+2} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ z_L & z_{L+1} & z_{L+2} & \cdots & z_N \end{bmatrix}$$

is of full row rank, $\operatorname{rank}\left(\mathcal{H}_{[L,N]}(z)\right) = qL$.

$\mathcal{H}_{[L,N]}(z)$ is square or has more columns than rows: $(q+1)L \leq N + 1$

A finite number of good samples can describe all trajectories

Fundamental Lemma

Let $w^d = (u^d, y^d) \in \mathcal{B}|_N$ be an N-long trajectory of a controllable system of order n.
If the input u^d is persistently exciting of order $L+n$, then

- any L-long trajectory is a linear combinations of the columns of $\mathcal{H}_{[L,N]}(w^d)$
- for any $\alpha \in \mathbb{R}^{N-L+1}$, the vector $\mathcal{H}_{[L,N]}(w^d)\alpha$ is a (restricted) trajectory
- $\mathcal{B}|_L = \mathrm{image}\ \mathcal{H}_{[L,N]}(w^d)$

- Only sufficient condition
- Can be a combination of several trajectories: $\begin{bmatrix} \mathcal{H}_{[L,N]}(w^{d,1}) & \mathcal{H}_{[L,N]}(w^{d,2}) \end{bmatrix}$

Willems, Rapisarda, Markovsky, De Moor (2005) A Note on Persistency of Excitation, *in Sys. & Cont. Let.*

The lemma applies to a simple $n = 1$ example

$$\xrightarrow[u^d_1,\dots,u^d_6]{} \boxed{y_{k+1} = y_k + u_k + u_{k+1}} \xrightarrow[y^d_1,\dots,y^d_6]{}$$

Set $L = 2$ and $N = 6 \implies \alpha \in \mathbb{R}^5$. The input u^d must be PE of order $L + n = 3$.

$$\begin{bmatrix} u_1 \\ u_2 \\ y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} u^d_1 & u^d_2 & u^d_3 & u^d_4 & u^d_5 \\ u^d_2 & {}^{d}u_3 & u^d_4 & u^d_5 & u^d_6 \\ y^d_1 & y^d_2 & y^d_3 & y^d_4 & y^d_5 \\ y^d_2 & y^d_3 & y^d_4 & y^d_5 & y^d_6 \end{bmatrix} \alpha = \begin{bmatrix} \mathcal{H}_{[L,N]}(u^d) \\ \mathcal{H}_{[L,N]}(y^d) \end{bmatrix} \alpha$$

(u, y) – any trajectory

The data-driven representation allows for control design

$$u_1^d, \dots, u_6^d \longrightarrow \boxed{y_{k+1} = y_k + u_k + u_{k+1}} \longrightarrow y_1^d, \dots, y_6^d$$

$$\begin{bmatrix} u_{ini} \\ y_{ini} \\ u \\ y \end{bmatrix} = \begin{bmatrix} u_{k-1} \\ y_{k-1} \\ u_k \\ y_k^r \end{bmatrix} = \begin{bmatrix} u_1^d & u_2^d & u_3^d & u_4^d & u_5^d \\ y_1^d & y_2^d & y_3^d & y_4^d & y_5^d \\ u_2^d & u_3^d & u_4^d & u_5^d & u_6^d \\ y_2^d & y_3^d & y_4^d & y_5^d & y_6^d \end{bmatrix} \alpha = \begin{bmatrix} U_p \\ Y_p \\ U_f \\ Y_f \end{bmatrix} \alpha$$

Open-loop control

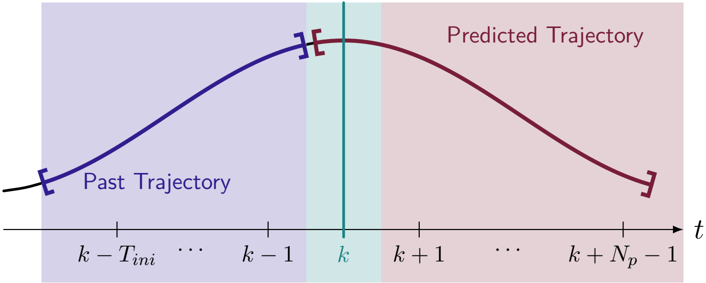
Given T_{ini}-long initial trajectory (u_{ini}, y_{ini}) and N_p-long desired trajectory y^r, find u such that the whole trajectory $w = (\bar{u}, \bar{y}) \in \mathcal{B}|_{L=T_{ini}+N_p}$, where $\bar{u} = \begin{bmatrix} u_{ini} \\ u \end{bmatrix}$, $\bar{y} = \begin{bmatrix} y_{ini} \\ y^r \end{bmatrix}$.

We construct the Data-driven LQ

Given the dataset w^d, the initial (past) trajectory (u_{ini}, y_{ini}), and the references r_u, r_y,

$$\min_{u,y,\alpha} \|y - r_y\|_Q + \|u - r_u\|_R$$

$$\text{subject to} \quad \begin{bmatrix} u_{ini} \\ y_{ini} \\ u \\ y \end{bmatrix} = \begin{bmatrix} U_p \\ Y_p \\ U_f \\ Y_f \end{bmatrix} \alpha$$



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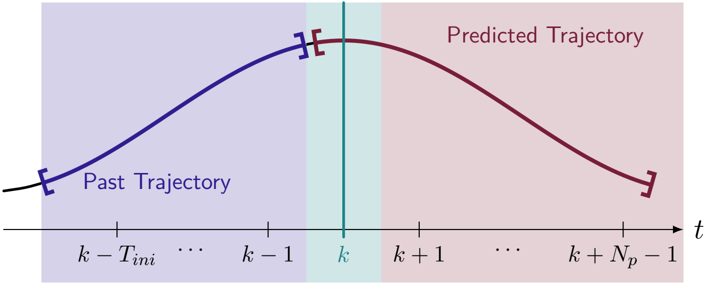
Noise?

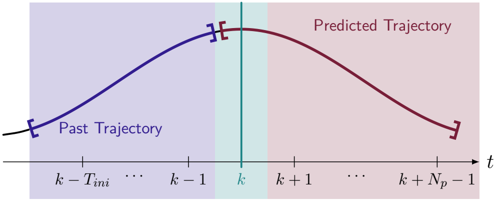


DeePC applies to noised problems

$$\min_{\substack{u,\,y\\ \alpha,\,\sigma}} \|y - r_y\|_Q^2 + \|u - r_u\|_R^2 + \lambda_\sigma \|\sigma\|_2^2 + \lambda_\alpha h(\alpha)$$

$$\text{subject to} \quad \begin{bmatrix} u_{ini} \\ y_{ini} + \sigma \\ u \\ y \end{bmatrix} = \begin{bmatrix} U_p \\ Y_p \\ U_f \\ Y_f \end{bmatrix} \alpha \text{ and } (u, y) \in \mathbb{U} \times \mathbb{Y}$$

- the slack variable σ
- regularization function $h(\alpha)$, e.g., $h(\alpha) = \|\alpha\|_2^2$
- feasible sets $\mathbb{U}$ and $\mathbb{Y}$
- tuning parameters λ_σ and λ_α



Coulson, Lygeros, Dörfler (2019) Data-enabled predictive control: In the shallows of the DeePC, *in ECC*

The next question is the DDPC for nonlinear systems

1. What are the behavioral approach and DDPC/DeePC for LTIs?

2. How can it be applied to nonlinear systems and what is the dataset management problem?
Examples included

3. What is the proposed solution?

The Lemma can be adapted to affine system

We linearize $x_{k+1} = f(x_k) + Bu_k$ and $y_k = h(x_k) + Du_k$

Linear system (at an equilibrium):

$$x_{k+1} = Ax_k + Bu_k$$
$$y_k = Cx_k + Du_k$$

Lemma: if u^d is PE of order $L + n$, then (u, y) is a trajectory $\Leftrightarrow \exists \alpha$ s.t.

$$\begin{bmatrix} u \\ y \end{bmatrix} = \begin{bmatrix} \mathcal{H}(u^d) \\ \mathcal{H}(y^d) \end{bmatrix} \alpha$$

Affine system:

$$x_{k+1} = Ax_k + Bu_k + e$$
$$y_k = Cx_k + Du_k + r$$

Lemma: if u^d is PE of order $L + n + 1$, then (u, y) is a trajectory $\Leftrightarrow \exists \alpha$ s.t.

$$\begin{bmatrix} u \\ y \end{bmatrix} = \begin{bmatrix} \mathcal{H}(u^d) \\ \mathcal{H}(y^d) \end{bmatrix} \alpha \text{ and } \sum \alpha_i = 1$$

DeePC can be applied to nonlinear systems via linearization

Given the dataset (u^d, y^d), at each step k solve (with the intermediate setpoints u^s, y^s)

$$\min_{\substack{\bar{u},\bar{y}\\ \alpha,\sigma \\ u^s, y^s}} \sum_{i=T_{ini}+1}^{L} \|\bar{y}_i - y^s\|_Q^2 + \|\bar{u}_i - u^s\|_R^2 + \|y^s - r_y\|_S^2 + \lambda_\sigma\|\sigma\|_2^2 + \lambda_\alpha\|\alpha\|_2^2$$

subject to $\bar{u} \in \mathbb{U}$, $\bar{y} \in \mathbb{Y}$ and

Lemma

$$\begin{bmatrix} \bar{u} \\ \bar{y} + \sigma \\ 1 \end{bmatrix} = \begin{bmatrix} \mathcal{H}(u^d) \\ \mathcal{H}(y^d) \\ \mathbb{1}^\top \end{bmatrix} \alpha$$

Initial conditions

$$\bar{u}_{[1,T_{ini}]} = u_{[k-T_{ini},k-1]}$$
$$\bar{y}_{[1,T_{ini}]} = y_{[k-T_{ini},k-1]}$$

Terminal constrains

$$\bar{u}_{[L-T_f+1,L]} = \mathbb{1} \otimes u^s$$
$$\bar{y}_{[L-T_f+1,L]} = \mathbb{1} \otimes y^s$$

and apply $u_k = \bar{u}_{T_{ini}+1}$. Here $\bar{u} \in \mathbb{R}^{n_u L}$ and $L = T_{ini} + N_p + T_f$.

Berberich, Köhler, Müller, Allgöwer (2022) Linear tracking MPC for nonlinear systems—Part II: The data-driven case, *in TAC*

DeePC can be applied to nonlinear systems via linearization

Given the dataset (u^d, y^d), at each step k solve (with the intermediate setpoints u^s, y^s)

$$\min_{\substack{\bar{u},\bar{y}\\ \alpha,\sigma\\ u^s,y^s}} \sum_{i=T_{ini}+1}^{L} \|\bar{y}_i - y^s\|_Q^2 + \|\bar{u}_i - u^s\|_R^2 + \|y^s - r_y\|_S^2 + \lambda_\sigma\|\sigma\|_2^2 + \lambda_\alpha\|\alpha\|_2^2$$

subject to $\bar{u} \in \mathbb{U}$, $\bar{y} \in \mathbb{Y}$ and

Lemma

$$\begin{bmatrix} \bar{u} \\ \bar{y}+\sigma \\ 1 \end{bmatrix} = \begin{bmatrix} \mathcal{H}(u^d) \\ \mathcal{H}(y^d) \\ \mathbb{1}^\top \end{bmatrix} \alpha$$

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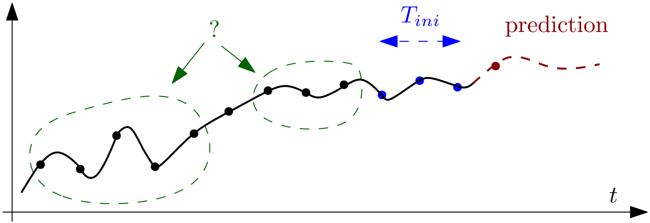
Berberich, Köhler, Müller, Allgöwer (2022) Linear tracking MPC for nonlinear systems—Part II: The data-driven case, *in TAC*

Offline data cannot be used anymore

Lemma

$$\begin{bmatrix} \bar{u} \\ \bar{y} + \sigma \\ 1 \end{bmatrix} = \begin{bmatrix} \mathcal{H}(u^d) \\ \mathcal{H}(y^d) \\ \mathbb{1}^\top \end{bmatrix} \alpha$$



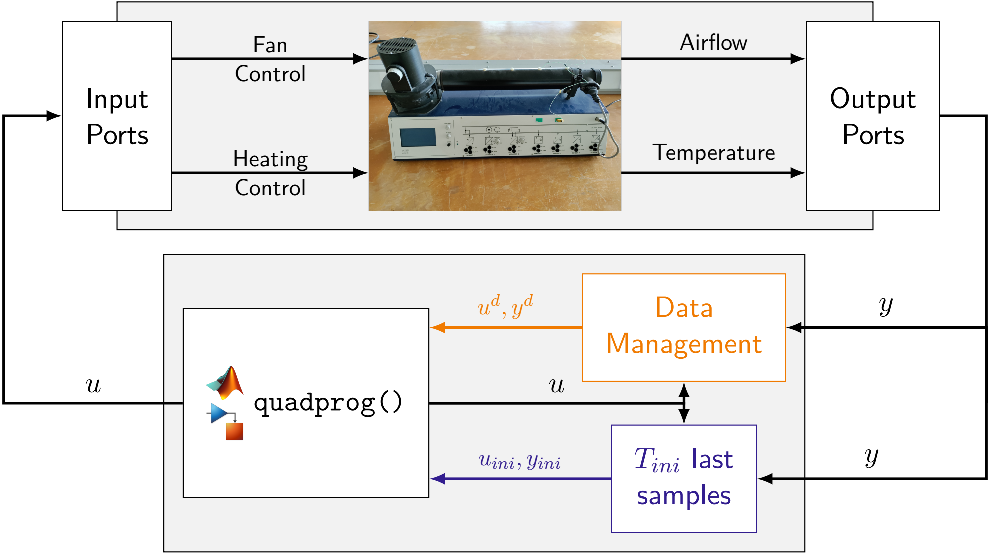
How to populate u^d and y^d?

- No offline data for linearized systems $\longrightarrow$ use the past samples of the same trajectory
- DDPC is based on the Lemma $\longrightarrow$ data must be sufficiently exciting
- Linearization point changes $\longrightarrow$ data must be recent

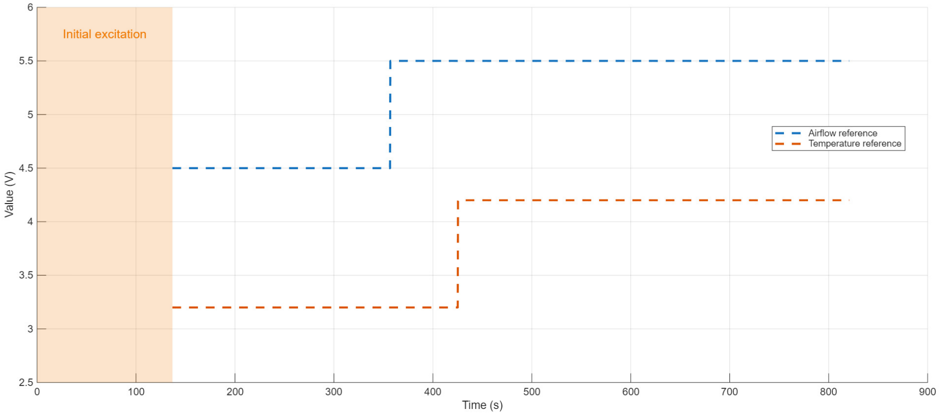
The problem: recent vs exciting

Only some heuristics are available

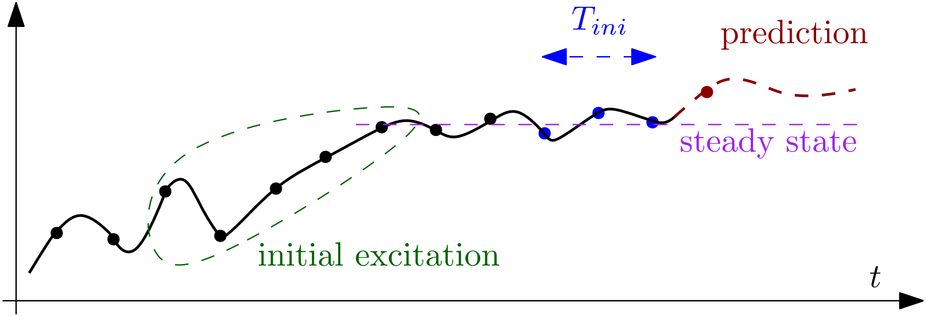
We make experiments on the heater-blower system



The experiment consist of the initial excitation and step references

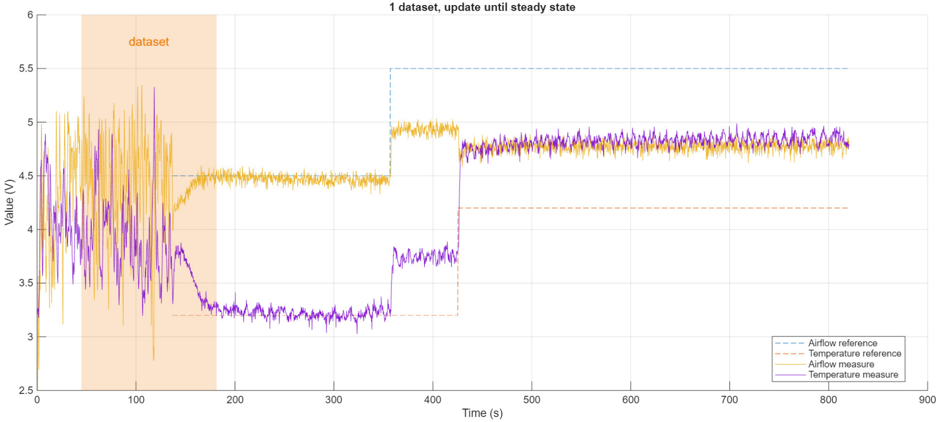


We can use the initial excitation until the steady-state

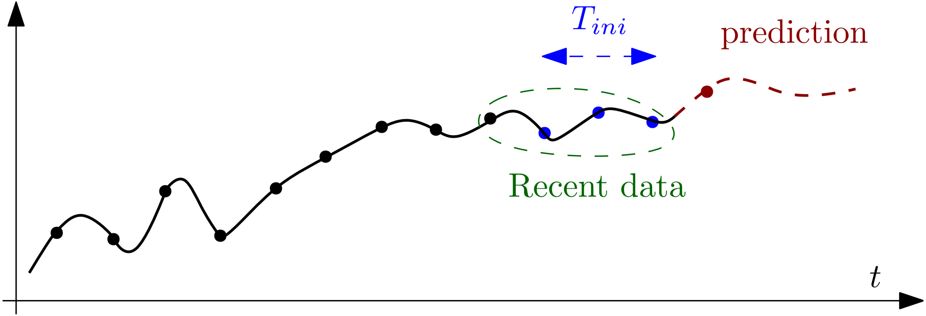


We stop updating (u^d, y^d) once the steady-state is achieved, $\|y - y^s\| \leq \epsilon$

This heuristic does not track the changes

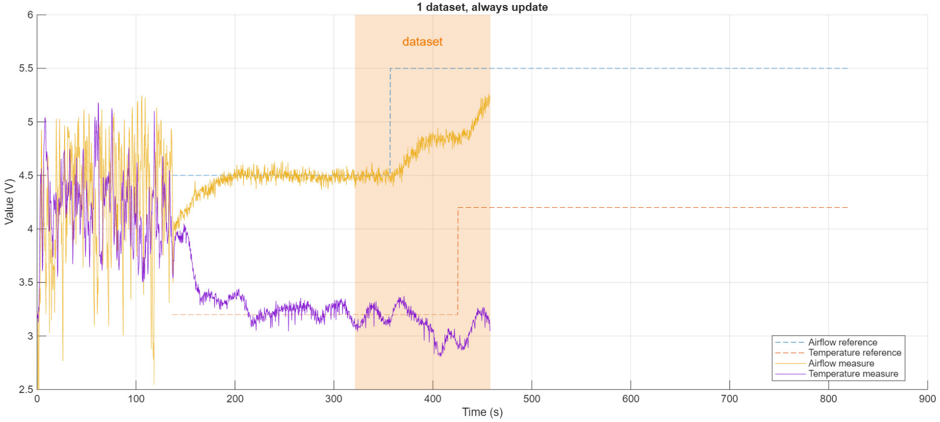


We can always use the recent data



We always use the most recent data as (u^d, y^d)

This heuristic does not work

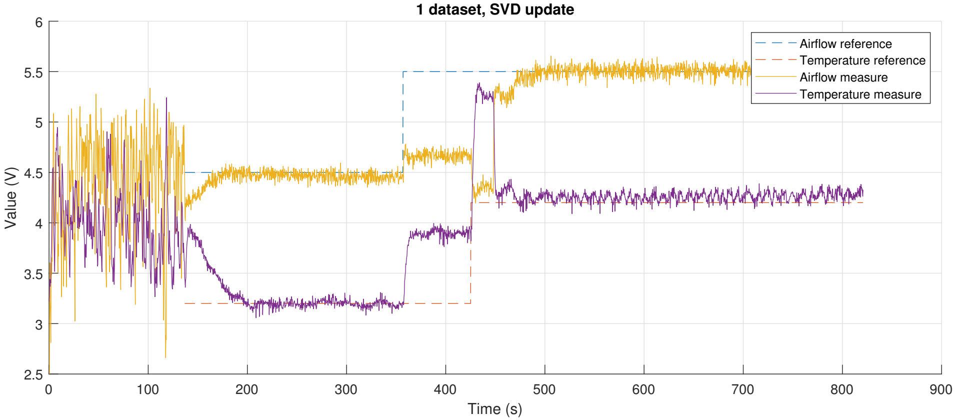


We can update only when the new data is exciting

We can use the smallest singular value $\sigma_{min}\left(\mathcal{H}(u)\right)$ as a measure of excitation.

1: Collect the initial exciting dataset (u^d, y^d)
2: **for** each time step t **do**
3: construct the candidate dataset $(\widetilde{u}^d, \widetilde{y}^d))$ from the recent measurements
4: **if** $\sigma_{min}(\mathcal{H}(\widetilde{u}^d)) \geq \bar{\sigma}$ **then** update the exciting dataset:
5: $(u^d, y^d) \leftarrow (\widetilde{u}^d, \widetilde{y}^d)$
6: **end if**
7: **end for**

This heuristic also has tracking problems



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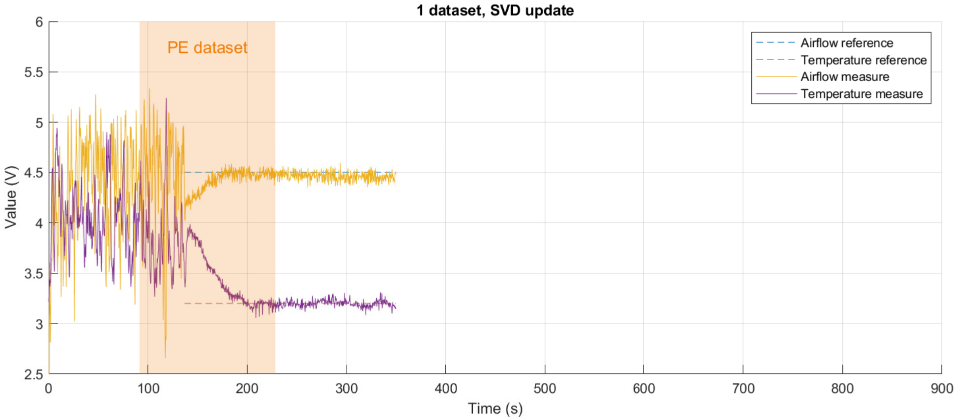

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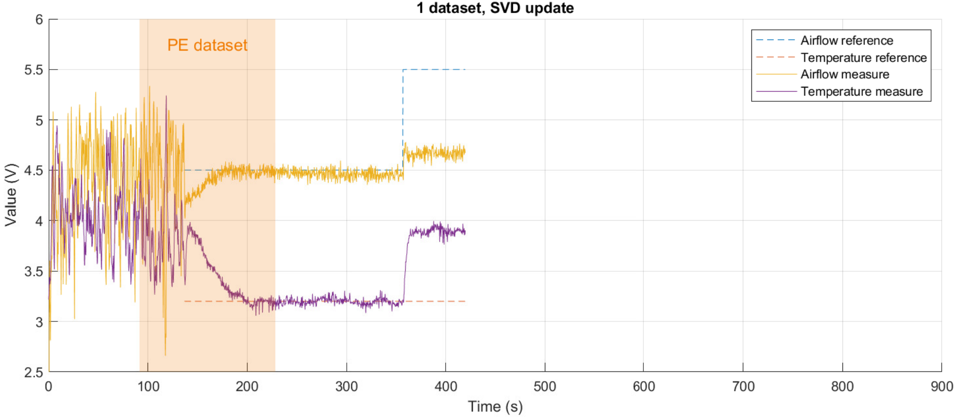

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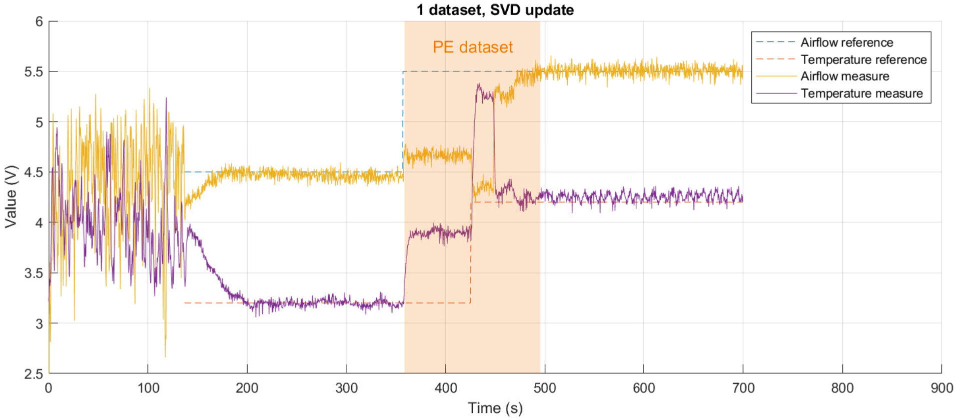


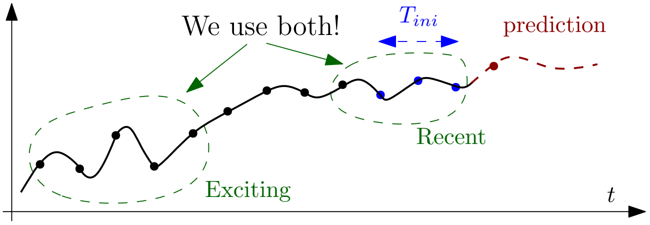
We propose another solution

1. What are the behavioral approach and DDPC/DeePC for LTIs?

2. How can it be applied to nonlinear systems and what is the dataset management problem?
Examples included

3. What is the proposed solution?

Two datasets can be used simultaneously



We can combine two datasets:

- the recent one to represent the linearization point, $(u^{d,r}, y^{d,r})$
- the past one to preserve the excitation, $(u^{d,e}, y^{d,e})$
- the past one can be updated, e.g., via the singular value criteria

How can we enforce such a structure?

Two datasets can be used simultaneously

Two datasets: the recent $(u^{d,r}, y^{d,r})$ with N_r samples, and the exciting $(u^{d,e}, y^{d,e})$ with N_e samples.

Lemma (Structured Datasets)

(u, y) is an L-long trajectory of

$$x_{k+1} = Ax_k + Bu_k + e$$
$$y_k = Cx_k + Du_k + r$$

if and only if there exist $\alpha_r \in \mathbb{R}^{N_r-L+1}$ and $\alpha_e \in \mathbb{R}^{N_e-L+1}$ such that

$$\begin{bmatrix} u \\ y \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} \mathcal{H}_{[L,N_e]}(u^{d,e}) & \mathcal{H}_{[L,N_r]}(u^{d,r}) \\ \mathcal{H}_{[L,N_e]}(y^{d,e}) & \mathcal{H}_{[L,N_r]}(u^{y,r}) \\ \mathbf{0} & \mathbb{1}^\top \\ \mathbb{1}^\top & \mathbf{0} \end{bmatrix} \begin{bmatrix} \alpha_e \\ \alpha_r \end{bmatrix}$$

Moreover, the constants e and r are encoded in $(u^{d,r}, y^{d,r})$ only.

But do we have any stability guarantee?

The stability analysis is adapted for the proposed dataset structure

Theorem (Stability analysis)

- Given reasonable assumptions on the nonlinear system,
- using the proposed structured dataset management strategy,
- limiting the distance from y to both datasets,
- and assuming a lower bound on the excitation level of the coupled datasets,

the trajectory converges into a vicinity of a (suitable) reference.

The proof is based on the original proof for nonlinear DeePC

Faye-Bedrin, Chauchat, Aranovskiy, Bourdais (*cond. accepted, 2025*) Structured Dataset Management for Data-Enabled Predictive Control of Nonlinear Systems, *in TCST*

We can track step-wise changes of the reference

Corollary (Step-wise reference changes)

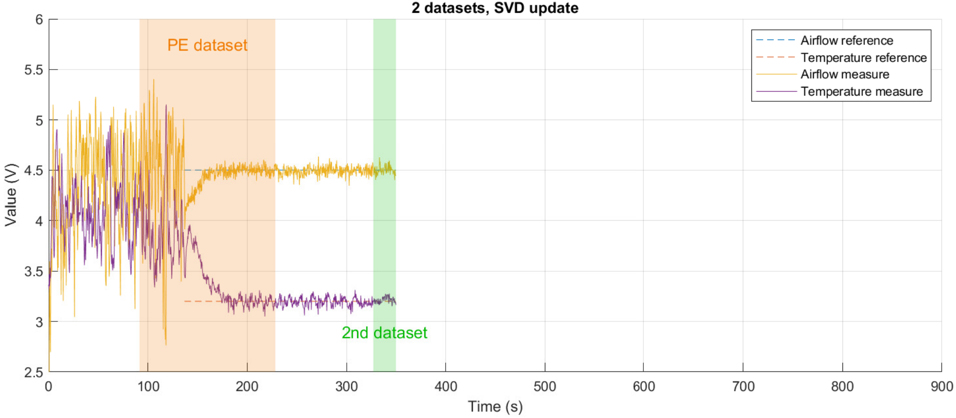
Assuming that the new reference remains close enough

- to the exciting dataset $y^{d,e}$
- and to the past reference,

the stability analysis applies.

- Either the exciting dataset of updated often enough
- or the changes in the reference are small

It works!



2 datasets, SVD update
PE dataset
2nd dataset
Airflow reference
Temperature reference
Airflow measure
Temperature measure
Value (V)
Time (s)
6
5.5
5
4.5
4
3.5
3
2.5
0
100
200
300
400
500
600
700
800
900

It works!

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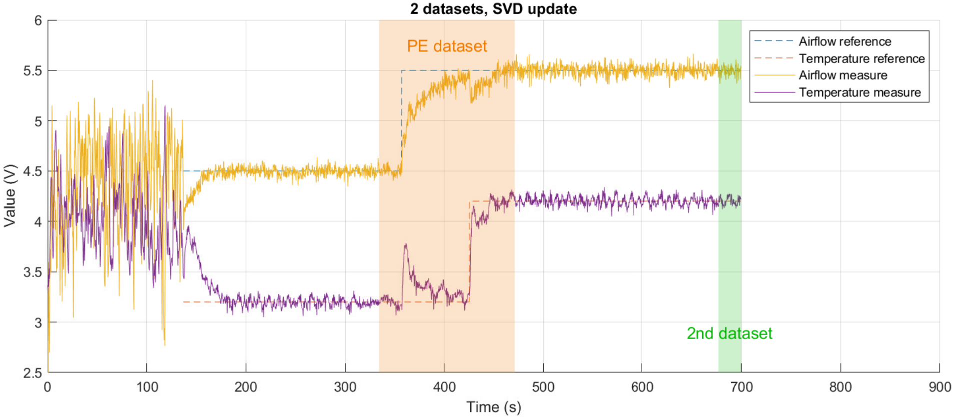
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900

Sometimes we need more than a “good dataset” assumption

The take-away №1

Maintaining a relevant dataset in DDPC for nonlinear systems is a nice question

Conclusion

- The behavioral approach + the Fundamental Lemma gave rise to the DDPC/DeePC
- DDPC can be extended to nonlinear systems, but requires online data collection
- The dataset management becomes crucial for performance
- The proposed structured dataset approach handles contradicting requirements (*confirmed by experiments*)
- Possible research directions: nonlinear systems, adaptive control and online data processing, merging with offline models, computational efficiency and recursive formulations, ...

The take-away №2

DDPC is an interesting, actively developing field with numerous open problems!

Thank you!